

Answers to Written Re-exam Economics Winter 2020-2021

## **Advanced Economics of the Environment and Climate Change**

Date: 18.02.2020 (9:00-12:30)

Answers only in English.

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# Optimal unilateral climate policy and border carbon adjustments

**(indicative weight: 3/4)**

*Generally about this exercise:* This exercise employs a modified version of the model presented in Lecture 19 and Exercise Class 14. The model is simplified in three ways: (1) there are two instead of  $N$  consumption goods, (2) consumers do not consume fossil fuels directly, and (3) there is no carbon leakage through the international market for fossil fuels. The objective of the government is, however, changed. Carbon leakage does not result in a direct cost for the government. Instead, the government has a binding emission constraint.

Consider a small open economy with two production sectors: light,  $l$ , and heavy,  $h$ , manufacturing. There are two types of consumption goods: one produced by each sector. Consumers derive utility from the consumption of these two consumption goods.

In the following description of the economy, units are chosen such that one unit of fossil fuel consumption leads to one unit of (carbon) emissions.

The utility level of the representative consumer is given by:  $u(c_l, c_h)$ , where  $c_l$  and  $c_h$  measure consumption of good  $l$  and  $h$ , respectively. The utility function is strictly increasing and concave in both arguments.

The representative consumer maximizes utility given its budget constraint and taking prices, aggregate variables, and policies as given. The budget constraint is given by:

$$I = c_l(p_l + t_l) + c_h(p_h + t_h),$$

where  $I$  is income,  $p_l$  and  $p_h$  are the international exogenous prices of consumption good  $l$  and  $h$ , and the domestic prices of good  $l$  and  $h$  are  $(p_l + t_l)$  and  $(p_h + t_h)$ . The domestic price of good  $l$  ( $h$ ) equals the international price of good  $l$  ( $h$ ) plus the tariff on good  $l$  ( $h$ ). This tariff is denoted  $t_l$  ( $t_h$ ).

Production in both sectors requires a fossil fuel input. Each sector is represented by a single firm that maximizes profits taking prices, aggregate variables, and policies as given.

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Profits in the two sectors,  $\pi_l$  and  $\pi_h$ , are given by:

$$\begin{aligned}\pi_l &= f_l(e_l)(p_l + t_l) - e_l(p_e + \tau_l), & f'_l(e_l) > 0, & \quad f''_l(e_l) < 0, \\ \pi_h &= f_h(e_h)(p_h + t_h) - e_h(p_e + \tau_h), & f'_h(e_h) > 0, & \quad f''_h(e_h) < 0,\end{aligned}$$

where  $f_l(e_l)$  and  $f_h(e_h)$  are the production of consumption good  $l$  and  $h$ ,  $e_l$  and  $e_h$  measure the input of fossil fuels in sector  $l$  and  $h$ , and  $\tau_l$  and  $\tau_h$  are sector specific carbon taxes.

Note that  $t_l$  ( $t_h$ ) is an export subsidy if the economy is a net exporter of good  $l$  ( $h$ ). Meanwhile, it is a tariff on imports if the economy is a net importer of good  $l$  ( $h$ ).

There is no domestic production of fossil fuels, and thus, all fossil fuels are imported. Trade must balance, implying that the value of net imports equals zero:

$$p_l m_l + p_h m_h + p_e (e_l + e_h) = 0, \quad m_l = c_l - f_l(e_l), \quad m_h = c_h - f_h(e_h),$$

where  $m_l$  and  $m_h$  are net imports of good  $l$  and  $h$ .

The government keeps a balanced budget. The entire tax revenue is transferred to the representative consumer through a lump-sum transfer,  $T$ :

$$T = \tau_l e_l + \tau_h e_h + t_l m_l + t_h m_h.$$

The representative consumer has two sources of income: (1) profits from the domestic firms, and (2) the lump-sum transfer from the government. Thus, the representative consumer's income is given by:

$$I = T + \pi_l + \pi_h.$$

Carbon leakage occurs through changes in trade patterns. The amount of carbon leaking to the foreign economy when the value of net imports increases is given by:

$$\begin{aligned}\mathcal{L}_l &= (m_l - m_{0,l}) L_l, & L_l > 0, \\ \mathcal{L}_h &= (m_h - m_{0,h}) L_h, & L_h > 0,\end{aligned}$$

where  $m_{0,l}$  and  $m_{0,h}$  are the net imports of good  $l$  and  $h$  without regulation, and  $L_l$  and  $L_h$  are constant leakage rates associated with imports. Specifically,  $L_l$  and  $L_h$  measure the

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increase in foreign emissions caused by a unit increase in net imports of good  $l$  and  $h$ .

Finally, to avoid corner solutions, it is assumed that:

$$\lim_{c_l \rightarrow 0} u'_l(c_l, c_h) = \infty, \quad \lim_{c_h \rightarrow 0} u'_h(c_l, c_h) = \infty,$$
$$\lim_{e_l \rightarrow 0} f'_l(e_l) = \infty, \quad \lim_{e_h \rightarrow 0} f'_h(e_h) = \infty,$$

where  $u'_l(c_l, c_h)$  and  $u'_h(c_l, c_h)$  denote the partial derivative of  $u(c_l, c_h)$  with respect to  $c_l$  and  $c_h$ , respectively. These technical assumptions ensure that the representative consumer always demands both types of goods, and there will be domestic production of both goods in a market equilibrium.

### Question 1.1

What carbon leakage channels are not present in the model? Your answer should give a short description of each of these channels.

*Answer to Question 1.1:* The model features leakage through the trade channel. It may be argued that three leakage channels are missing from the model.

Firstly, there is no leakage through the international market for fossil fuels. As a reduction of domestic fossil fuel consumption reduces the international price of fossil fuels, foreign economies will increase their fossil fuel consumption in response. Incorporating this leakage channel into the model would require that leakage also depended directly on the domestic fossil fuel consumption.

Secondly, leakage may also occur through international climate policies and agreements. One example is an international cap-and-trade system, where a unilateral emission reduction leads to a reduction in the emission allowance price, as the allowance supply is fixed. Thus, emissions in the foreign economies that are part of the system increase. Note that it can be argued that this leakage channel is captured by the model, as the foreign economy is not explicitly modelled. Thus, the leakage rates  $L_l$  and  $L_h$  may partly reflect that the foreign economy is restricted by international climate agreements.

Thirdly, leakage may occur through technological development. A unilateral tightening of the climate policy may direct research toward more environmentally friendly technologies. These technologies may also be available to and employed by the foreign economy. This could lead to a reduction in foreign emissions: resulting in a negative leakage effect.

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## Question 1.2

Characterize the optimal behavior of consumers and firms in the market equilibrium of this economy given some policy (that is some values of  $\tau_l$ ,  $\tau_h$ ,  $t_l$ , and  $t_h$ ).

*Answer to Question 1.2:* The representative consumer maximizes utility with respect to  $c_l$  and  $c_h$  subject to the budget constraint taking prices, aggregate variables, and policies as given. The Lagrangian associated with the representative consumer's problem is:

$$\mathcal{L} = u(c_l, c_h) + \gamma [I - c_l(p_l + t_l) - c_h(p_h + t_h)],$$

where  $\gamma$  is the shadow price of income. The first-order conditions are:

$$u'_l(\cdot) = \gamma(p_l + t_l) \quad \text{and} \quad u'_h(\cdot) = \gamma(p_h + t_h).$$

This implies that the marginal rate of substitution is:

$$\frac{u'_l(\cdot)}{u'_h(\cdot)} = \frac{p_l + t_l}{p_h + t_h}.$$

Each representative firm solves the problem:

$$\max_{e_i} f_i(e_i)(p_i + t_i) - e_i(p_e + \tau_i), \quad i = \{l, h\}.$$

The first-order condition implies that:

$$f'_i(e_i)(p_i + t_i) = p_e + \tau_i.$$

The above constitutes a sufficient answer to the question. One could add that the market equilibrium is characterized by the following system of equations:

$$\frac{u'_l(c_l, c_h)}{u'_h(c_l, c_h)} = \frac{p_l + t_l}{p_h + t_h},$$

$$f'_l(e_l)(p_l + t_l) = p_e + \tau_l,$$

$$f'_h(e_h)(p_h + t_h) = p_e + \tau_h,$$

$$p_l(c_l - f_l(e_l)) + p_h(c_h - f_h(e_h)) + p_e(e_l + e_h) = 0.$$

The system consists of 4 equations and 4 unknowns ( $e_l$ ,  $e_h$ ,  $c_l$ ,  $c_h$ ). The trade balance

constraint (the last equation) follows directly from the income constraint of the representative consumer.

The government wants to maximize the representative consumer's utility, but it also wants to reduce global emissions. Specifically, the government has the emission target,  $\bar{E}$ , which consists of domestic emissions and carbon leakage:

$$\bar{E} = e_l + e_h + \mathcal{L}_l + \mathcal{L}_h. \quad (1)$$

### Question 1.3

Show that the optimal allocation given the government's emission target, (1), implies that:

$$\begin{aligned} u'_l(\cdot) &= \lambda p_l + \eta L_l, & u'_h(\cdot) &= \lambda p_h + \eta L_h, \\ \lambda f'_l(e_l) p_l + \eta f'_l(e_l) L_l &= p_e \lambda + \eta, & \text{and} & \quad \lambda f'_h(e_h) p_h + \eta f'_h(e_h) L_h = p_e \lambda + \eta, \end{aligned}$$

where  $\lambda$  is the shadow cost (negative shadow price) of imports, and  $\eta$  is the shadow price of emissions. Explain these equations intuitively. *Hint: the government wants to maximize the representative consumer's welfare under the emission constraint and the terms-of-trade constraint.*

*Answer to Question 1.3:* The Lagrangian associated with the government's problem is given by:

$$\begin{aligned} \mathcal{L} &= u(c_l, c_h) - \lambda [p_l (c_l - f_l(e_l)) + p_h (c_h - f_h(e_h)) + p_e (e_l + e_h)] \\ &\quad + \eta [\bar{E} - e_l - e_h - ((c_l - f_l(e_l)) - m_{0,l}) L_l - ((c_h - f_h(e_h)) - m_{0,h}) L_h], \end{aligned}$$

The first-order conditions are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_i} = u'_i(\cdot) - \lambda p_i - \eta L_i = 0 &\quad \Rightarrow \quad u'_i(\cdot) = \lambda p_i + \eta L_i \\ \frac{\partial \mathcal{L}}{\partial e_i} = \lambda f'_i(e_i) p_i + \eta f'_i(e_i) L_i - p_e \lambda - \eta = 0 &\quad \Rightarrow \quad \lambda f'_i(e_i) p_i + \eta f'_i(e_i) L_i = p_e \lambda + \eta, \end{aligned}$$

where  $i = \{l, h\}$ .

In optimum, the social marginal cost of consuming one additional unit of good  $i = \{l, h\}$  must equal the social marginal benefit. The same holds for the social marginal cost and

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benefit of using one additional unit of the fossil fuel in sector  $i = \{l, h\}$ .

The first two equations reflect that in optimum, the marginal benefit of consuming good  $i = \{l, h\}$  must equal the marginal cost of doing so. The marginal benefit is the marginal utility of consuming good  $i$ . The marginal cost consists of two terms: (1) an import cost, and (2) a leakage cost.

To understand the marginal cost expression, consider the effect of increasing the consumption of good  $i$  by one unit on imports. Holding the fossil fuel use - and thereby domestic emission - constant, the consumption of one additional unit of good  $i$  results in an increase in imports by  $p_i$  units. This has two costs. The first is given by the term  $\lambda p_i$ , as  $\lambda$  is the shadow cost of imports. Basically, an increase in the consumption of good  $i$  must be offset by the reduction in net imports of other goods. Thus, increasing imports has a cost, and the unit cost of increasing imports is given by the shadow cost  $\lambda$ . Second, increasing net imports of good  $i$  results in an increase in foreign production and thereby emissions. In other words, there will be carbon leakage associated with an increase in net imports of good  $i$ . This has a cost, as emissions elsewhere must be reduced to ensure the government's emission target. The unit value of emissions is  $\eta$ , and thus, the leakage cost is given by  $\eta L_i$ .

The next two equations state that the social marginal benefit from fossil fuel use in sector  $i = \{l, h\}$  must equal the social marginal cost. Note that given the consumption of good  $i$ , using one additional unit of fossil fuel results in a decrease in net imports by  $f'_i(e_i)$  of good  $i$ . The marginal benefit consists of two terms: (1) the value of the relaxation of the trade balance constraint given by  $\lambda f'_i(e_i) p_i$ , and (2) the value of reduced leakage from international trade given by  $\eta f'_i(e_i) L_i$ . Note that  $\eta$  is the shadow price of global emissions and global emissions are reduced by  $f'_i(e_i) L_i$  through this leakage channel.

The marginal cost is also divided into two terms. The first term,  $p_e \lambda$ , reflects the trade balance cost of importing one additional unit of fossil fuel. The second term,  $\eta$ , reflects the cost of the emission stemming from using one additional unit of fossil fuels.

### Question 1.4

Characterize the optimal climate policy (the optimal choice of  $t_l$ ,  $t_h$ ,  $\tau_l$ , and  $\tau_h$ ) given the emission target (1). Briefly explain the regulation in words.

*Answer to Question 1.4:* Comparing the market allocation with the optimal allocation, it is clear that the following equations must hold for the optimal solution to be implemented in

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the market economy:

$$\frac{\lambda p_l + \eta L_l}{\lambda p_h + \eta L_h} = \frac{t_l + p_l}{t_h + p_h} \quad \text{and} \quad \frac{p_e \lambda + \eta}{\lambda p_i + \eta L_i} = \frac{\tau_i + p_e}{p_i + t_i},$$

where the first equation equals  $u'_l(\cdot)/u'_h(\cdot)$  and the second equals  $f'_i(e_i)$ .

The first equation holds if:

$$t_i + p_i = \lambda p_i + \eta L_i \quad \Leftrightarrow \quad t_i = (\lambda - 1)p_i + \eta L_i.$$

Substituting this equation into the second expression:

$$\frac{p_e \lambda + \eta}{\lambda p_i + \eta L_i} = \frac{\tau_i + p_e}{\lambda p_i + \eta L_i} \quad \Leftrightarrow \quad p_e \lambda + \eta = \tau_i + p_e \quad \Leftrightarrow \quad \tau_i = (\lambda - 1)p_e + \eta.$$

The optimal regulation scheme involves a uniform emission tax on domestic emissions, and border carbon adjustments on both types of goods, which depends on international prices and sector-specific leakage. Thus, the border carbon adjustments differ between sectors.

### Question 1.5

The government now moves to a purely domestic emission target. Specifically, the government wants to ensure that:

$$\tilde{E} = e_l + e_h. \tag{2}$$

Characterize the optimal climate policy, when the emission constraint is given by (2). Comment on your findings.

*Answer to Question 1.5:* This corresponds to the case  $L_l = L_h = 0$ . Thus, one can use the results from Question 1.3 and 1.4 to answer the question. The optimal allocation requires that:

$$\frac{u'_h(\cdot)}{u'_l(\cdot)} = \frac{p_h}{p_l}.$$

To implement this in the market economy, we need  $t_l = t_h = 0$ . According to the expression from Question 1.4, this implies that:  $\lambda = 1$ . Hence, the optimal domestic carbon tax,  $\tau^*$ , is



given by:

$$\tau_i = \eta \equiv \tau^*.$$

Thus, the optimal regulation requires a uniform tax on all domestic emissions. Intuitively, this ensures an equalization of marginal abatement costs. It is therefore not possible to reduce emission reduction costs further by changing the emission reduction burdens between sectors.

Alternatively, one can find the optimal allocation for this particular problem and derive the policy that implements this allocation.

The Lagrangian associated with the government's problem is now given by:

$$\begin{aligned} \mathcal{L} = & u(c_l, c_h) - \lambda [p_l(c_l - f_l(e_l)) + p_h(c_h - f_h(e_h)) + p_e(e_l + e_h)] \\ & + \eta [\tilde{E} - e_l - e_h], \end{aligned}$$

The first-order conditions are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_i} = u'_i(\cdot) - \lambda p_i = 0 & \Rightarrow u'_i(\cdot) = \lambda p_i, \\ \frac{\partial \mathcal{L}}{\partial e_i} = \lambda f'_i(e_i) p_i - p_e \lambda - \eta = 0 & \Rightarrow \lambda f'_i(e_i) p_i = p_e \lambda + \eta, \end{aligned}$$

where  $i = \{l, h\}$ .

From here, the reasoning is almost the same as stated above. To implement the optimal allocation in the market economy, we need  $t_l = t_h = 0$ , as the optimal allocation requires that  $u'_l(\cdot)/u'_h(\cdot) = p_l/p_h$ . Using the expression for  $f'_i(e_i)$  from the market equilibrium and the optimal allocation:

$$\frac{p_e \lambda + \eta}{\lambda p_i} = \frac{p_e + \tau_i}{p_i} \Leftrightarrow \tau_i = \frac{\eta}{\lambda}.$$

Thus, the optimal regulation requires a uniform tax on all domestic emissions. The intuition is given above.

Consider the situation from before, where the government's emission target is given by (1). Now assume that the government cannot implement the border carbon adjustment mechanism without facing a costly trade war.

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## Question 1.6

Show that - in this particular setting - the government can implement the optimal allocation under emission constraint (1) using sector-specific carbon taxes and good-specific consumption taxes. Explain the intuition. Please denote the consumption taxes by  $\tilde{t}_l$  and  $\tilde{t}_h$ . *Hint: you need to remove the border carbon adjustments from the model and implement the consumption taxes appropriately.*

*Answer to Question 1.6:* The budget constraint of the representative consumer is now given by:

$$I = c_l(p_l + \tilde{t}_l) + c_h(p_h + \tilde{t}_h),$$

and the profit functions are given by:

$$\pi_l = f_l(e_l)p_l - e_l(p_e + \tau_l) \quad \text{and} \quad \pi_h = f_h(e_h)p_h - e_h(p_e + \tau_h).$$

Solving the representative consumer's problem, one obtains the following condition from the first-order conditions:

$$\frac{u'_h(\cdot)}{u'_l(\cdot)} = \frac{p_h + \tilde{t}_h}{p_l + \tilde{t}_l}.$$

The first-order condition of firm  $i$  amounts to:

$$f'_i(e_i)p_i = \tau_i + p_e.$$

Comparing these conditions to the optimality conditions from Question 1.3, we see that to implement the optimal allocation in the market economy, the policy instruments need to be adjusted according to:

$$p_i + \tilde{t}_i = \lambda p_i + \eta L_i \quad \Leftrightarrow \quad \tilde{t}_i = (\lambda - 1)p_i + \eta L_i.$$
$$f'_i(e_i) = \frac{\lambda p_e + \eta}{\lambda p_i + \eta L_i} = \frac{\tau_i + p_e}{p_i} \quad \Leftrightarrow \quad \tau_i = \frac{\eta(p_i - p_e L_i)}{\lambda p_i + \eta L_i}.$$

Intuitively, the sector-specific carbon taxes can control the emission and thereby production levels in the two sectors. Meanwhile, the consumption taxes can control the representative consumer's consumption through a change in relative prices. Controlling both domestic pro-

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duction and consumption levels implies that the government can also control net imports and thereby carbon leakage. The government, therefore, has sufficient instruments to implement the optimal allocation even without border carbon adjustments.

## **Exercise 2: Environmental regulation under uncertainty**

### **(indicative weight: 1/4)**

(Hint: You may provide purely verbal answers to the questions in this exercise, but you are also welcome to include equations if you find it useful)

Economists typically argue that a tax on pollution emission and a cap-and-trade system can implement the optimal emission level if there is no uncertainty about pollution abatement costs and pollution damages.

Assume that pollution abatement costs and pollution damage costs are convex. This implies that the marginal abatement cost is increasing in the emission reduction, and that marginal damages are increasing in the emission level.

### **Question 2.1**

Explain how the convexity of the cost functions are connected to the trade-off between a tax and a cap-and-trade system when there is uncertainty about pollution abatement costs.

*Answer to Question 2.1:* Starting with the pollution damages, varying emission levels result in a higher expected damage cost than a constant emission level, when the expected emission level is the same. This follows from Jensen's inequality, as the pollution damage costs are convex by assumption. Accordingly, since a cap-and-trade system ensures a fixed emission level, it results in a lower expected cost of pollution emission.

A similar intuition holds for the pollution abatement costs. A pollution tax holds the price and thereby the marginal abatement cost constant. In that sense, the instrument equalizes marginal abatement costs across periods (or draws depending on the narrative). This minimizes abatement costs. In contrast, the cap-and-trade system results in varying marginal abatement costs. Hence the price instrument results in a lower expected cost of pollution abatement.

Summing up, the price instrument results in lower expected abatement costs, while the

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cap-and-trade system results in a lower expected pollution damage cost. The trade-off between the price and quantity instrument is determined by the relative importance of these two cost types. And this is captured by the relative slope of the marginal abatement cost and marginal damage cost curves.

A steeper marginal damage cost curve implies that the cost of varying emission levels is higher, which favours the quantity instrument. A steeper marginal abatement cost curve favours the tax instrument, as this implies that a fixed emission level and thereby varying marginal abatement costs result in higher (expected) total abatement costs.

One could also emphasize that the cap-and-trade system becomes more attractive if a stock pollutant is considered, as current emissions affect pollution damages at later dates. This amplifies the cost of a varying emission level, as policy errors from one period spill into the next periods.

## Question 2.2

Assume that all countries in the world decide to price CO<sub>2</sub>e emissions to mitigate climate change. Discuss whether they should implement a tax or a cap-and-trade system.

*Answer to Question 2.2:* There are multiple ways to give a satisfactory answer to this question.

The basic trade-off between the price and quantity instrument is given in the answer to Question 2.1. Thus, this answer focuses on features specific to the climate change problem.

As a fraction of the CO<sub>2</sub> emitted today stays in the atmosphere for centuries, the choice between the tax and cap-and-trade system must account for the stock feature of the externality. Other greenhouse gasses decay faster or slower. But it does not seem reasonable to think of any of them as a flow pollutant.

Given that the problem is about regulating a stock pollutant, the cap-and-trade system becomes more attractive. This is because variations in the emission level caused by the tax instrument spill into subsequent periods, amplifying the pollution damage costs associated with the instrument.

However, it is sometimes argued that the marginal damage cost curve for greenhouse gasses is relatively flat. This feature of the problem makes the price instrument more attractive.

The trade-off also depends on the stock of greenhouse gasses in the atmosphere when the decision is made. The model analysis by Newell and Pizer (2003) shows that it may

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be beneficial to switch from a price to a quantity instrument at some point. Basically, the accumulation of greenhouse gasses in the atmosphere increases the marginal damage cost of emissions. This mechanism makes the quantity instrument increasingly more attractive over time.

In the end, there are arguments for and against the two instruments. One has to conduct an empirical assessment to figure out which instrument is preferred. In a simulation exercise, Newell and Pizer (2003) find that the price instrument is more attractive even when considering long time horizons.

## References

R. G. Newell and W. A. Pizer. Regulating stock externalities under uncertainty. *Journal of Environmental Economics and Management*, 45(2):416–432, 2003.